General Certificate of Education
Advanced Level Examination
January 2010

Mathematics (Pilot) XMCA2

Unit Core A2

Monday 25 January 2010 9.00 am to 11.30 am

For this paper you must have:
- a 16-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 2 hours 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is XMCA2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 125.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) (i) Sketch the graph of $y = x(5 - x)$.

(ii) On separate axes, sketch the graph of $y = |5x - x^2|$, indicating the coordinates of any points where the graph meets the coordinate axes. *(3 marks)*

(b) Solve the equation $|5x - x^2| = 6$. *(4 marks)*

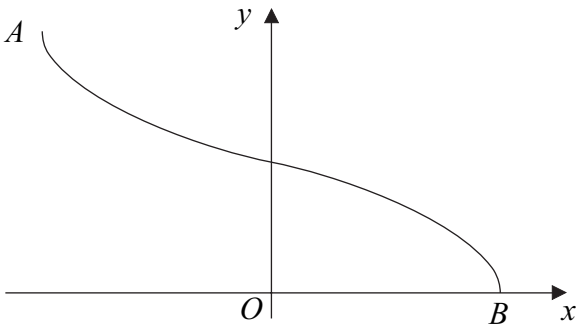
2 A curve is defined by the parametric equations

$$x = 3t + 2, \qquad y = 6t^2$$

(a) At the point P on the curve, $t = 1$. Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = ax + b$. *(6 marks)*

(b) Find, in the form $y = \mathrm{f}(x)$, a cartesian equation of the curve defined by the parametric equations $x = 3t + 2$ and $y = 6t^2$. *(2 marks)*

3 The sketch shows the graph of $y = \cos^{-1} x$, where y is in radians.



(a) Write down the coordinates of the points A and B, the end-points of the graph. *(2 marks)*

(b) (i) By drawing a suitable straight line on a copy of the above sketch, explain why the equation $\cos^{-1} x = 4x - 1$ has only one root. *(2 marks)*

(ii) Given that the root of the equation $\cos^{-1} x = 4x - 1$ is α, show that $0.5 < \alpha < 0.6$. *(2 marks)*

(c) Use the iteration $x_{n+1} = \dfrac{1 + \cos^{-1} x_n}{4}$ with $x_1 = 0.5$ to find the values of x_2, x_3 and x_4, giving your answers to three significant figures. *(3 marks)*

(d) Use Simpson's rule with five ordinates (four strips) to find an approximation to

$$\int_0^{0.8} \cos^{-1} x \, \mathrm{d}x$$

giving your answer to four decimal places. *(4 marks)*

4 (a) Sketch the graph of $y = 1 + \mathrm{e}^{2x}$, indicating the value of y where the graph crosses the y-axis. *(2 marks)*

(b) (i) Given that $y = \dfrac{\cos x}{1 + \mathrm{e}^{2x}}$, find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0$. *(4 marks)*

(ii) Write down the value of $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ when $x = 0$. *(1 mark)*

5 (a) Given that $\dfrac{6x^2+10}{(3x-1)(x+1)^2}$ can be expressed in the form $\dfrac{A}{3x-1}+\dfrac{B}{(x+1)^2}$, where A and B are integers, find the values of A and B. *(4 marks)*

(b) Hence find $\displaystyle\int_1^3 \frac{6x^2+10}{(3x-1)(x+1)^2}\,\mathrm{d}x$, giving your answer in the form $p\ln 2+q$. *(6 marks)*

6 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x)=\sqrt{x+4}, \quad \text{for } x \geqslant 0$$

$$\mathrm{g}(x)=\frac{1}{x+2}, \quad \text{for real values of } x,\ x \neq -2$$

(a) State the range of f. *(2 marks)*

(b) Find $\mathrm{gf}(x)$. *(1 mark)*

(c) The inverse of f is f^{-1}. Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

7 (a) The polynomial $\mathrm{p}(x)$ is defined by $\mathrm{p}(x)=30x^3-x^2-6x+1$.

(i) Find $\mathrm{p}\left(\frac{1}{3}\right)$. *(1 mark)*

(ii) Use the Factor Theorem to show that $2x+1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(iii) Write $\mathrm{p}(x)$ as a product of three linear factors. *(3 marks)*

(b) Given that $\tan 2\theta=\dfrac{2t}{1-t^2}$, where $t=\tan\theta$, show that the equation

$$\cot 2\theta = 18-15\sec^2\theta$$

can be written in the form

$$30t^3-t^2-6t+1=0 \qquad \textit{(4 marks)}$$

(c) Hence solve the equation

$$\cot 2\theta = 18-15\sec^2\theta$$

in the interval $0^\circ \leqslant \theta \leqslant 180^\circ$, giving all solutions to the nearest 0.1°. *(3 marks)*

8 (a) Find $\int 2x \ln x \, dx$. *(4 marks)*

(b) Use a suitable substitution to find $\int 4x^2\sqrt{2x+1} \, dx$. *(5 marks)*

9 A painting was valued at £ 8000 on 1 January 2002. A mathematical model assumes that t years later, the value, £ V, of the painting is increasing at a rate proportional to $V^{\frac{1}{3}}$.

(a) By forming and solving a differential equation, show that $3V^{\frac{2}{3}} = 2kt + 1200$, where k is a constant. *(6 marks)*

(b) On 1 January 2010, the painting is valued at £ 27 000. Use this mathematical model to find the value of the painting on 1 January 2015, giving your answer to the nearest £ 100. *(3 marks)*

10 (a) Find the binomial expansion of $(1+kx)^{-2}$ up to and including the term in x^3, where k is a constant. *(2 marks)*

(b) The binomial expansion in ascending powers of x of

$$(1+kx)^{-2} - (1+x)^n$$

is $6x^2 + px^3 + \ldots$.

(i) Show that $n = -2k$. *(2 marks)*

(ii) Given that $n < 0$, find the value of p. *(6 marks)*

11 The line l_1 through the point $A(1, 2, 3)$ has vector equation $\mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$.

The line l_2 has vector equation $\mathbf{r} = \begin{bmatrix} 24 \\ 9 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 12 \\ 3 \\ 4 \end{bmatrix}$.

(a) Prove that l_1 and l_2 intersect and find the coordinates of their point of intersection B. *(6 marks)*

(b) Prove that there does **not** exist a point P on the line l_2 such that triangle ABP is equilateral. *(4 marks)*

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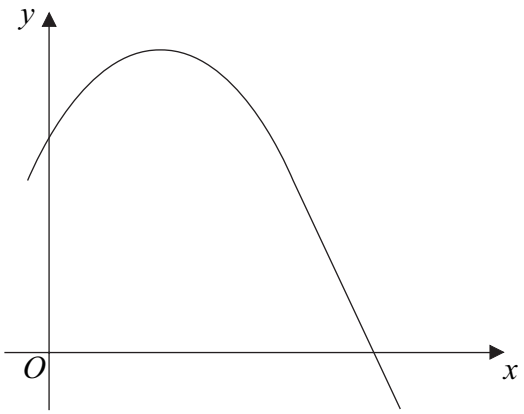
12 The diagram shows part of the curve C with equation $y = 8\cos x + 6\sin x$.



It is given that $8\cos x + 6\sin x$ can be expressed in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(a) State the value of R and the value of $\sin\alpha$. *(2 marks)*

(b) Hence describe a sequence of two geometrical transformations which maps the graph of $y = \cos x$ onto the graph of $y = 8\cos x + 6\sin x$. *(4 marks)*

(c) The region bounded by the curve C, the x-axis, the y-axis and the line $x = \alpha$ is rotated through 2π radians about the x-axis. Show that the volume of the solid formed is $2\pi(25\alpha + p)$, where p is a constant to be found. *(8 marks)*

13 A curve has equation $3y^3 - 6xy + 2x^3 = k$, where k is a constant. Show that, if $k < -\frac{4}{3}$, the curve has no stationary points. *(9 marks)*

END OF QUESTIONS

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